Alternating current

Ac voltage applied to a resistor

A time-varying voltage that varies like a sine function is called AC voltage.

The current driven by an AC voltage is called AC current.

AC is preferred to DC mainly because AC voltages can be easily and efficiently converted from lower to higher voltages and vice versa using transformers.

In a pure resistor, the current flowing through the resistor is in phase with the potential difference across it.

The ratio of the instantaneous voltage and instantaneous current in a resistor is constant.

AC- power, RMS current, voltage and phasors

RMS current is defined as the root of the mean of the square of current.

For an AC current: $I_{\text{rms}} = I_m/\sqrt{2} = 0.707 \times I_m$

RMS voltage or effective voltage: $V_{\text{rms}} = V_m/\sqrt{2}$

The average power dissipated by any form of AC current in a resistor, $P = I_{\text{rms}}^2 R$
A phasor is a representation of a sine wave whose amplitude, phase and frequency are time invariant.

**AC voltage applied to an inductor**

An inductor is an electrical component that can store energy in the magnetic field created by a varying current flowing through it.

Any change in current through an inductor induces an emf across it, which is proportional to the rate of change of current.

The power supplied to an inductor on an average in an AC circuit over one complete cycle is zero.

\[
(P) = -\frac{i_m v_m}{2} \quad (\sin 2 \omega t) = 0
\]

**AC voltage applied to a capacitor**

A capacitor is an electronic component that can store an electrostatic charge. It can be similar to a tiny battery.

A simple capacitor consists of two conducting plates separated by an insulating material called a dielectric.

When the capacitor is fully charged, the charge on the capacitor opposes any further current flow through it and the current in the circuit falls to zero in a steady state.
Capacitive reactance, \( X_c \), is the inverse of the product of angular frequency, \( \omega \), and capacitance, \( C \),

\[ X_c = \frac{1}{\omega C} \]

Power supplied to a capacitor on average in a circuit is zero.

\[ (P) = \frac{v_m i_m}{2} \sin (2\omega t) = 0 \]

The cyclic discharging and charging of the capacitor in reverse polarities continues as long as AC current is established in the circuit.

**LCR circuit – phasor diagram solution**

Instantaneous circuit: \( i = i_m \sin(\omega t + \phi) \)

Maximum current: \( i_m = \frac{V_m}{\sqrt{R^2 + \left(X_C - X_L\right)^2}} = \frac{V_m}{Z} \)

Phase difference: \( \tan \phi = \frac{X_c - X_L}{R} \)

\[ \phi = \tan^{-1} \left( \frac{X_c - X_L}{R} \right) \]

**LCR circuit – analytical solution**

Instantaneous voltage: \( v = V_m \sin \omega t \)
Maximum voltage: \( V_m = i_m Z \)

Instantaneous current: \( i = i_m \sin (\omega t + \varphi) \)

Maximum current: \( i_m = q_m \omega \) or
\[
    i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}
\]

**LCR circuit – resonance**

Resonance is a phenomenon observed in systems that tend to oscillate at some fixed frequency under the influence of an external periodic force required to sustain its oscillation.

Condition for resonance in an LCR circuit: \( X_c = X_L \)

Resonant frequency: \( \omega_0 = \frac{1}{\sqrt{LC}} \)

Resonance is possible only in a circuit having both capacitive and inductive components as the condition, \( X_c = X_L \) needs to be satisfied.

Bandwidth of the circuit: \( 2\Delta \omega = \omega_0/Q \)

Sharpness of resonance: \( \omega_0 / 2\Delta \omega = \omega_0 L / R \)

Quality factor: \( Q = \omega_0 L / R = 1 / \omega_0 CR \)

A higher value of the quality factor leads to a sharper resonance as the bandwidth decreases.
Power in AC circuit

Power in AC circuit: \( P_{av} = V_{rms} I_{rms} \cos \phi \) \( P_{av} = I_{rms}^2 Z \cos \phi \)

Power factor: \( \cos \phi \)

The current in a purely inductive or purely capacitive circuit, where the power factor is zero, is known as wattles current.

LC oscillations

The phenomenon of LC oscillations can be observed when a charged capacitor is placed in series with an inductor in a circuit.

In an LC oscillator, the charging and discharging of a capacitor is oscillatory in nature.

For an LC oscillation: \( q = q_m \cos(\omega_0 t) \) \( i = i_m \sin(\omega_0 t) \)

Where:

\( i_m = \omega_0 q_m \)

\( q = \) instantaneous charge on the capacitor

\( i = \) instantaneous current in the circuit

\( q_m = \) maximum charge on the capacitor in a steady state

\( \omega_0 = \) angular frequency

\( t = \) time

The total energy in a LC circuit is always a constant.

\( U_{total} = q_m^2 / 2C \)
**Transformers**

A transformer is a device used to increase or decrease the voltage of an alternating current. The transformers work on the principle of mutual induction. According to the principle of mutual induction, when two coils are placed close to each other and if one of the coils carries a time varying current, a current is induced in the other coil.

For an ideal transformer the ratio of output voltage to input voltage is equal to the ratio of input current to output current which is equal to the ratio of number of turns in secondary coil to that in primary coil.

\[
\frac{V_s}{V_p} = \frac{i_p}{i_s} = \frac{N_s}{N_p}
\]

If the secondary coil has a greater number of turns than the primary, the transformer is called a step-down transformer.

If the secondary coil has a lesser number of turns than the primary, the transformer is called a step-down transformer.